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# Gas Lift Optimization: Using Least Square Method and Solver

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*Research Article*

# Gas Lift Optimization: Using Least Square Method and Solver

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**ABSTRACT**

**In this study a model was developed using least square method. This model was tested for reliability and it was found to be reliable for prediction. We now use this model for optimization of oil production.The following reports was obtained after optimization.**
***Answer report:*** **Oil produced original value was 3471,251,263.00STB/Day and the final value was 1347,252, 519.25STB/Day.**
***Sensitivity Report:*** **Allowable increase in liquid injected was 32.19893491bbl/day and Allowable decrease in liquid injected was 0.591566837bbl/day. Also Allowable increase in water cut was 1E+30bbl and Allowable decrease in water cut was 96.59680522bbl/day.**
***Limit Report:*** **liquid injected lower limit -1.03228E-10bbl and target result was not available and. The water cut target was 0. 1347252519bbl.**
**It is evident that a real optimization model that can optimize oil production for different amount of liquid injected and water cut has been developed. Such a model can provide the oil industries in the Country important information about the sensitivity of the optimal solution.**

**Keywords: Model,optimization,Limits,Sensitivity,Production.**

## INTRODUCTION

Gas Lift is one of a number of processes used to artificially lift oil or water from wells where there is insufficient reservoir pressures to produce the well[1,2]. The process involves injecting gas through the tubing-casing annulus. Injected gas aerates the fluid to reduce its density; the formation pressure is then able to lift the oil column and forces the fluid out of the wellbore. Gas may be injected continuously or intermittently, depending on the producing characteristics of the well and the arrangement of the gas-lift equipment[3,4,5]. Being somehow an ancient tool with an age of over a century, gas-lift is though still a challenging problem when overall optimization is the concern. When injection gas is of a limited supply the problem is finding the best gas allocation scheme. However, there are increasingly emerging cases in certain geographic localities where the gas supplies are usually unlimited. The optimization problem then totally travels to the wellbore and completion string and fully engages with multiphase flow concepts.

Various research persons who work in petroleum field wrote on gas lift system since past up to present[6]. Optimization problems are real world problems we encounter in many areas such as mathematics, engineering, science, business and economics. In these problems, we find the optimal, or most efficient, way of using limited resources to achieve the objective of the situation. This may be maximizing the production, minimizing the injected fluid, minimizing the total distance travelled or minimizing the total time to complete a project. For the given problem, we formulate a mathematical description called a mathematical model to represent the situation[7].

In this paper a model was developed using a 11 years production history and the model was tested for effectiveness and efficiency. Optimization was carried out using Excel solver.

## AIMS AND OBJECTIVE

The aim of this project is:
To develop a model for prediction and carry out optimization of oil production

**Scope and Limitation**

This project will be confined to development of a model using 10-year production data.

**Research Design**

In this work, the amount of oil produced annually for ten years (1998-2007) were analyzed by a graph of oil produced against each variable, $q_l$ and $q_w$. It was found from the graphs, that there is linear relationship between each variable and product. With this insight, a least square method of regression was adopted. The regression equation in study is given below[8].

$q_0 = b_1 + b_2q_l + b_3q_w$------------------------------------------------------------1.0

The normal equations for equation 1.0 are as follow:

$\sum q_o = nb_1 + b_2\sum q_l + b_3\sum q_w$----------------------------------1.1
$\sum q_oq_l = b_1\sum q_l + \sum q_l^2 + b_3\sum q_wq_l$-------------------------------1.2
$\sum q_oq_w = b_1\sum q_w + \sum q_wq_w + b_3\sum q_w^3$----------------------------1.3

Computed data are fixed into the normal equations generated from equation 1 above. The resulting equations reduced to a matrix form as shown below.

$30735.66 = b_1\ 11 + b_2\ 28813.84 + b_3\ 11.480823$ ---------------------1.4
$90407117.49 = b_1\ 28813.84 + b_2\ 91418701.8 + b_3\ 25338.27497$-----1.5
$24544.01184 = b_1\ 11.480823 + b_2\ 25338.27497 + b_3\ 60.10575813$ ---1.6

The regression parameters ($b_1$, $b_2$ and **$b_3$) were obtained as follow:**

$b_1 = 1347.251263$ , $b_2 = 0.5915666171$ **$b_3$=**98.37358741

Therefore

$q_0 = 1347.251263 + 0.591666171q_l - 98.37358741q_w$ ------------------------1.7

**Model Testing**

The model developed was used to generate values given by $q_o$. The residuals were computed and Durbin Watson test for auto correlation is given as;

$$D.W = \frac{\sum_{1-2}^{n}\left(\ell_t - \ell_{t-1}\right)^2}{\sum_{t-1}^{n}\ell_t^2}$$

In equation 3.1 above, $\ell_t$ = residual which is the difference between the actual Y value and that obtained with the model. $\left(\ell_t - \ell_{t-1}\right)$ is the difference between the residual and a previous residual.

**Hypothesis**

The test for autocorrelation using the Durbin Watson D.W., statistic is unique, in that there are certain range of D.W values for which we can neither reject. $H_0$ no correlation exist nor fail to reject it. In this work hypothesis are;

$H_0$: that no autocorrelation exist in error function.
$H_1$: that there is that exist in error function.

**Decision**

If computed D.W.≥ $d_L$ reject $H_0$
If computed D.W.≤ $d_L$ accept $H_0$
Equation 1.7 is referred to as **OBJECTIVE FUNCTION** of the optimization model.
Equation 1.7 was then used to generate estimate values for $q_0$ which was used for the durbin Watson test for correlation, see Table1.0.

Table 1.0: **The Durbin Watson Test For Correlation**

| TIME (date) d/m/y | ANNUAL OILPROD-UCTION | ESTIMATED OIL PRODUCTION $q_0$ ESTIMATE | RESIDUAL OIL PRODUC-TION $e\tau$ | $((e_\tau - e_{\tau-1})^2)$ | $e_\tau^2$ |
|---|---|---|---|---|---|
| 8/28/1998 | 1948.22 | 1871.951442 | 76.268558 | 0 | 5816.892939 |
| 8/28/1999 | 2182.47 | 2424.842814 | -242.372814 | 101532.324 | 59230.32659 |
| 8/28/2000 | 2293.44 | 2685.636485 | -392.96485 | 22677.96131 | 154421.3733 |
| 8/28/2001 | 2643.84 | 2887.078264 | -243.238264 | 22418.05056 | 59164.85307 |
| 8/28/2002 | 2723.84 | 2938.699716 | -214.859716 | 805.3419866 | 46164.69756 |
| 8/28/2003 | 2497.53 | 2816.279353 | -318.749353 | 10793.05668 | 101601.15 |
| 8/28/2004 | 3235.52 | 3225.083191 | 10.436809 | 108363.5293 | 108.9269821 |
| 8/28/2005 | 3069.32 | 3149.309221 | -79.989221 | 8176.866902 | 6398.275476 |
| 8/28/2006 | 4208.22 | 3833.105419 | 375.114581 | 207119.4706 | 140710.9489 |
| 8/28/2007 | 3512.05 | 3416.884974 | 95.165026 | 78371.75334 | 9056.382174 |
| 8/28/2007 | 2421.21 | 2776.404342 | -355.194342 | 202823.5603 | 126163.0206 |
| | | | Σ | 762356.915 | 708836.8476 |

NOTE that:
$H_0$ : That no correlation exist in error function G
$H_1$ : That there is correlation that exist in error function from Durbin Watson test

$$D.W = \frac{\sum_{1-2}^{n}(e_\tau - e_{\tau-1})^2}{\sum_{\tau-1}^{n} e_\tau^2}$$

$$D.W = \frac{762,356.915}{708,836.8476}$$
$$= 1.0755$$

From Data

n = no of well(s) production = 11
R = 2
for 5% of Production
$d_{lower}$
$d_L$ = 0.758
$d_{upper}$
$d_u$ = 1.604
Since D.W = 1.0755

That is $d_L$< 1.0755 < du
then D.W< du
$H_0$ is rejected

This implies that correlation exist in the residual $e_t$. That is we have comprehensively considered all requisite variables necessary to study the problem.

Therefore the model developed is valid and it can be used to compare actual maximum oil produced with estimated base on the least square, and also deduce maximum oil produced level for ranged liquid injected rate and the water cut produce.

Presently, a model has been developed and tested to be adequate. To estimate maximum oil produced using liquid injected rate and water cut for different number of times for eleven times. Hence, we can now use solver as in Table 2.0 to optimize oil production.

Table 2.0: Gaslift Optimization

| | **Decision Variables** | $q_l$ | $q_w$ | | | |
|---|---|---|---|---|---|---|
| values of decision variables | 1347251263 | 2123.6 | 0 | | | |
| Oil produced | $1,347,252,519.25 | $0.59 | $98.37 | | | |
| | | | | | | |
| | **CONSTRAINTS** | $q_l$ | $q_w$ | | | |
| | | 0.5 | 1.5 | ≤ | 1061.8 | 1061.8 |
| | | 2.3 | 3.3 | | 8086.38 | 4884.28 |
| | | 1 | 0 | | 0 | 2123.6 |
| | | 0 | 1 | | 0 | 0 |

**RESULTS**

The following are the reports of the optimization

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Table 3.0: Answer Report

| **Cell** | **Name** | **Original Value** | **Final Value** |
|---|---|---|---|
| $B$4 | Oil produced Decision Variables | 1,347,251,263.00STB/Day | 1,347,252,519.25STB/Day |

| **Cell** | **Name** | **Original Value** | **Final Value** |
|---|---|---|---|
| $C$3 | values of decision variables ql | 0 | 2123.6 |
| $D$3 | values of decision variables qw | 0 | 0 |

Constraints

| **Cell** | **Name** | **Cell Value** | **Formula** | **Status** | **Slack** |
|---|---|---|---|---|---|
| $G$7 | ≤ | 1061.8 | $G$7<=$F$7 | Binding | 0 |
| $G$8 | | 4884.28 | $G$8<=$F$8 | Not Binding | 3202.10045 |
| $G$9 | | 2123.6 | $G$9>=$F$9 | Not Binding | 2123.6 |
| $G$10 | | 0 | $G$10<=$F$10 | Binding | 0 |

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**Table 4.0: Sensitivity Report**

**Adjustable Cells**

| | | **Final** | **Reduced** | **Objective** | **Allowable** | **Allowable** |
|---|---|---|---|---|---|---|
| **Cell** | **Name** | **Value** | **Production** | **Coefficient** | **Increase** | **Decrease** |
| $C$3 | values of decision variables ql | 2123.6 | 0 | 0.596046448 | 32.19445546 | 0.596046448 |
| $D$3 | values of decision variables qw | 0 | 0 | 98.37150574 | 1E+30 | 96.58336639 |

Constraints

| | | **Final** | **Shadow** | **Constraint** | **Allowable** | **Allowable** |
|---|---|---|---|---|---|---|
| **Cell** | **Name** | **Value** | **Production** | **R.H. Side** | **Increase** | **Decrease** |
| $G$7 | ≤ | 1061.8 | 1.192092896 | 1061.8 | 696.1087935 | 1061.8 |
| $G$8 | | 4884.28 | 0 | 8086.38045 | 1E+30 | 3202.10045 |
| $G$9 | | 2123.6 | 0 | 0 | 2123.6 | 1E+30 |
| $G$10 | | 0 | 96.58336639 | 0 | 707.8666667 | 889.4723472 |

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Table 5.0: Limits Report

| | **Target** | |
|---|---|---|
| **Cell** | **Name** | **Value** |
| $B$4 | Oil produced Decision Variables | 1,347,252,519.25STB/Day |

| | **Adjustable** | | | **Lower** | **Target** | | **Upper** | **Target** |
|---|---|---|---|---|---|---|---|---|
| **Cell** | **Name** | **Value** | | **Limit** | **Result** | | **Limit** | **Result** |
| $C$3 | values of decision variables ql | 2123.6 | | -1.03228E-10 | 1347251263 | | 2123.6 | 1347252519 |
| $D$3 | values of decision variables qw | 0 | | #N/A | #N/A | | 0 | 1347252519 |

**DISCUSSION**

The model developed is adequate and it can be use for prediction and for optimization.

**The Answer Report**

A solver was used to compute programme solution using the objective function. Analysis shows that input of values of decision variable will generate required oil production as optimal solution. That is, once the constraints are entered and the target and changing cells are selected, then solver will generate the optimal solution (if one exists). The first value from cell is 1061.8, the total optimization obtained for first constraints $0.5q_{\ell} + 1.5q_w$ and the last cell value, 0. is the value of $0q_l + 1q_w$, which is the left-hand side of the last constraint in the optimal solution. The slack for each constraints is the amount by which the right-hand side differs from the actual value.

The status column indicates whether a constraint is Binding or Not Binding. Not Binding means that the optimized corner point is not located on the constraint line. Binding means that the optimal corner point is located on that constraint line (see Table3.0).

### Sensitivity Report

The sensitivity report provides information on how changes in the parameters of the objective function affects the optimization solution. This is important when the parameters are not the same or when they can be changed. Due to these change in parameters, the sensitivity of the objective function provides an allowable increase and allowable decrease, indicates how far the right-hand side of each constraints can be changed before the optimal solution to the model shifts to a different corner point.

A shadow price literally is a price in the shadows. It is not a price that must be paid since it indicates the change in the objective function assuming the right-hand side of a constraint is increased by 1 unit and everything else remains the same.

The first shadow production is the value 1.183133674, which is the increase in the objective function when 1061.8 is changed by 1 unit. See Table 4.0 for sensitivity report.

### Limiting Report

The limiting report considers boundary values for which the objective function operates. The constraints for the model produce certain target values. Since the objective function constraints have both upper limits constraints and lower limit constraints, results are obtained for lower limit with target results; also upper limits generates target results for the range of values(See Table5.0).

## CONCLUSION

With the analysis carried out above and the subsequent discussion, it is evident that a real optimization model that can optimize oil production for different amount of liquid injected and water cut has been developed. Such a model can provide the oil industries in the Country important information about the sensitivity of the optimal solution.

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