Title: Statistical Magnitude Analysis and Distance Determination of the Nearby Stars

By

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Research Article

Statistical Magnitude Analysis and Distance Determination of the Nearby Stars

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ABSTRACT

Goals of this paper are: to provide some basic descriptive statistical parameters for the apparent and absolute magnitudes of the nearby stars of spectral type G5V. Second, to establish the frequency functions \( \Phi(M) \) and \( \Psi(m) \) of the absolute and apparent magnitudes for these stars. Third, to compute the distance \( r \) of these stars as a system assuming that they scatter around a mean absolute magnitude in a Gaussian distribution. The accuracy of the numerical results is satisfactory.

Keywords: Distance determinations, Spectral types, Frequency functions, Statistical magnitude analysis.

INTRODUCTION

G stars, of which the Sun is one, are yellow with temperatures around 5000 to 6000 K. Another example of the G class is Alpha Centauri A. The spectra of these stars are solar–type spectra, with Call lines extremely strong, neutral metals prominent, ions weaker, band (CH) strong and H lines are weaker [Robinson,1985]. As for the late spectral type stars, G class is less concentrated in the Milky Way than the early types[e.g. Scheffler and Elsässer 1988], as an example, the number of the G type stars per square degree up to the limiting magnitude \( m_p = 8.5 \) is about 0.4, the corresponding number of the A type stars at the latitude 5°.

G5 stars of luminosity class V, are denoted as G5V (main sequence ), their effective temperature about 5600K, bolometric correction -0.1 [Mihalas and Binney,1981], absolute magnitude +5.1,mass:
\[ \log(M / M_{\text{Sun}}) = -0.03 \],radius: \[ \log(R / R_{\text{Sun}}) = -0.03 \] , luminosity : \[ \log(L / L_{\text{Sun}}) = -0.1 \] and mean density (g cm\(^{-3}\)) : \[ \log \rho = 0.2 \] [Zombeck,1990]

In the present paper, we consider G5V stars and perform the following:

• First, to provide some basic descriptive statistics parameters for the apparent and absolute magnitudes of these stars.
• Second, to establish the frequency functions \( \Phi(M) \) and \( \Psi(m) \) of the absolute and apparent magnitudes for these stars.
• Third, to compute the distance \( r \) of these stars as a system assuming that they scatter around a mean absolute magnitude in a Gaussian distribution.

The accuracy of the numerical results is satisfactory in that, the percentage error between \( r \) and the mean value is less than 2.5%.
BASIC MATERIALS

2.1 Data

- The source of data is (V/70 A) Near by stars, preliminary 3rd. version (Gliese+1991) of (I/239) Hipparcos and Tycho Catalogues (ESA 1997)
- In Table I of Appendix A, the data for some G5V stars are listed. The columns of the table have the following meaning:

▲ Column 1: Name with the following acronyms:
  - G1 Gliese:
  - GJ Gliese& Jahreiss,AA&AS,38423(1979)
  - NN newly added stars (number added at CDS)

▲ Column 2: Vmag (unit mag): Apparent magnitude
▲ Column 3: B-V (unit mag): Color
▲ Column 4: U-B (unit mag): Color
▲ Column 5: plx (unit mas≡ milli-second of arc): Parallax
▲ Column 6: M_v (unit mag): Absolute visual magnitude

Appendix A: Input data

Table I: Data for some G5V near by stars

<table>
<thead>
<tr>
<th>Name</th>
<th>Vmag</th>
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<th>Mv</th>
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<td>mag</td>
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</table>
2.2 The distance equation

- The assumptions upon which the distance equation was derived were [Sharaf et al. 2003]

(1) All the members in a given cosmic group are at the same distance, \( r \) parsecs.

(2) The frequency function for the absolute magnitudes of the members is

\[
\Phi(M) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\left(M-M_0\right)^2/2\sigma^2}
\]

That is the members scatter around a mean absolute magnitude \( M_0 \) in a Gaussian distribution with dispersion \( \sigma \).

(3) The mean apparent magnitude \( \bar{m} \) of the members of the cosmic group and the limiting apparent magnitude \( m_1 \) are related through the quantity \( \alpha \) where

\[
\alpha = \frac{m_1 - \bar{m}}{\sigma}
\]

According to the above assumptions, the distance \( r \) and the frequency functions \( \Psi(m) \) of the apparent magnitude are given respectively by

\[
r = 10^{1+(m_1-M_0-\sigma y)/5}
\]

\[
\Psi(m) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(m+5-5\log r-M_0)^2/2\sigma^2}
\]

Where \( y \) is the solution of the transcendental equation:

\[
\Lambda(y) = y + e^{-y^2/2} \left\{ \frac{\pi}{2} \left[ 1 + \text{erf}\left(\frac{y}{\sqrt{2}}\right) \right] \right\}^{-1} - \alpha = 0,
\]

And, \( \text{erf}(z) \) is the error function defined by the integral

\[
\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt
\]

We call Equation (4) the distance equation.

- The relation between \( M_0 \) and \( \sigma \) is given by [Bok, 1937]

\[
M_0 = \bar{M} + 1.382 \sigma^2
\]

Where \( \bar{M} \) is the mean absolute magnitude of the stellar group.
2.3 *Empirical relation between* $M_V$ *versus* $(B-V)_0$

**Table II: Empirical relation between $M_V$ versus $(B-V)_0$**

<table>
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<th>$M_V$</th>
<th>$(B-V)_0$</th>
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</table>

- An empirical relation between the absolute visual magnitude $M_V$ versus $(B-V)_0$ is given in Mihalas and Binney, book [1981] in a tabular form (Table II of Appendix A).
- From this empirical relation, we deduce an interpolating polynomial between $(B-V)_0$ and $M_V$ in the form:

$$ (B-V)_0 = \sum_{j=1}^{20} q_j M_V \quad (7) $$

Where $q's$ are listed in Table III of Appendix A.

**Table III: The q's coefficients of Equation(7)**

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<th>$q_j$</th>
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NUMERICAL APPLICATION

3.1 Procedure

1- Compute the parallax \( p \) in seconds of arc from

\[ p = \text{plx} / 1000 \]

2- Compute for each star in the group \((B - V)_0\) from its given value of \( M_v \) using Equation ((7))

3- Compute the absorption \( A_v \) for each star of the group from

\[ A_v = 3 \left[ (B - V) - (B - V)_0 \right] \]

4- Compute the average \( \overline{A}_v \) of \( A_v \)

5- Correct the apparent magnitudes for absorption from

\[ V_{\text{mag}} = V_{\text{mag}} - A_v \]

6- Compute the average value \( \overline{M}_v \) of the absolute magnitudes \( M_v \)

7- Compute the average value \( \overline{r} \) of the individual distances \( r_j (r_j = 1 / p_j) \)

8- Compute the median \( r_{\text{median}} \) of the individual distances

9- Select start and end values of \( \sigma \), say \( \sigma_s \) and \( \sigma_e \) respectively. [In the present application, \( \sigma_s = 0.5 \) and \( \sigma_e = 2 \)]

10- Compute the optimum value of \( \sigma \) as follows

a- For \( \sigma = \sigma_s (0.01) \sigma_e \) perform the following calculations

- \( M_0 = \overline{M}_v + 1.382 \sigma^2 \)

- From the values of \( V_{\text{mag}}, M_0 \) and \( \sigma \) compute the distance \( r \) (in parsec) of the stellar group as obtained from the statistical method using the algorithm of Sharaf et al. (2003).

- Compute the percentage error \( f \) in the mean from \( f = | (r - \overline{r}) / \overline{r} | \times 100 \)

- Compute the percentage error \( g \) in the median from \( g = | (r - r_{\text{median}}) / r_{\text{median}} | \times 100 \)

b- Compute the value \( \sigma_1 \) that gives the minimum of the \( f \)'s values

c- Compute the value \( \sigma_2 \) that gives the minimum of the \( g \)'s values
d- Compute the optimum value of $\sigma$ as $\sigma = (\sigma_1 + \sigma_2)/2$.

11- Compute some basic descriptive statistics parameters for the apparent and absolute magnitudes of the stellar group.

12-Compute the measures of the central tendency for the individual distances $0$ of the stellar group, of these measures are:

▲ The Mean distance $\bar{r}$ (in parsec) of $r_j$
▲ The Mode of $r_j$
▲ The Median of $r_j$
▲ The Harmonic Mean of $r_j$
▲ The Geometric Mean of $r_j$

13-From the optimum values of $M_0$ and $\sigma$ compute the frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitudes [Equations (1) and (3)] of the group

3.2 Numerical Results

Numerical results are listed in Appendix B covering the following points:

* Statistics of the apparent and absolute magnitudes of the G5V group
* Graphical representations:

a- Histograms for the apparent and absolute magnitudes of the stellar group
b- Graphical representations for the percentage errors for the mean and median

* Optimum choice of $\sigma$
* Distance analysis
* The frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitudes for the G5V group

CONCLUSION

In concluding, we draw the attention that the aim of the present paper is three folds: First, to provide some basic descriptive statistical parameters for the apparent and absolute magnitudes of the nearby stars of spectral type G5V stars. Second, to establish the frequency functions $\Phi(M)$ and $\Psi(m)$ of the absolute and apparent magnitudes for these stars. Third, to compute the distance $r$ of these stars as a system assuming that they scatter around a mean absolute magnitude in a Gaussian distribution. The accuracy of the numerical results is satisfactory in that, the percentage error between $r$ and the mean value less than 2.5%. This percentage error indicates the applicability of the used method for distance determination in the ranges of the considered stellar group.

The analysis of the present paper will be extended for the stars of early spectral types and will constitute the task to which we shall consider soon.
REFERENCES


Appendix B: Output data

B.1: Statistics of the apparent and absolute magnitudes of the G5V stars

▲ The average:
For Apparent Magnitudes = 6.75649
For Absolute Magnitudes= 5.18667

▲ The median (central value):
For Apparent Magnitudes = 6.68
For Absolute Magnitudes= 5.17

▲ The mode:
For Apparent Magnitudes = {5.86,6.35,6.45,6.48,6.87,6.97}
For Absolute Magnitudes= {4.92,4.94,5.11,5.14,5.2}

▲ The geometric mean:
For Apparent Magnitudes = 6.702
For Absolute Magnitudes= 5.173

▲ The harmonic mean:
For Apparent Magnitudes = 6.64372
For Absolute Magnitudes= 5.15935

▲ The root mean square:
For Apparent Magnitudes = 6.80724
For Absolute Magnitudes= 5.20163

▲ The Quartiles:
For Apparent Magnitudes = {6.335,6.68,7.38}
For Absolute Magnitudes= {4.94,5.17,5.36}

▲ The range:
For Apparent Magnitudes = 4.34
For Absolute Magnitudes= 2.59

▲ The unbiased estimate of the variance:
For Apparent Magnitudes = 0.700666
For Absolute Magnitudes= 0.158212

▲ The maximum likelihood estimate of the variance:
For Apparent Magnitudes = 0.688374
For Absolute Magnitudes= 0.155436
The unbiased estimate of the variance of the sample mean:
For Apparent Magnitudes = 0.0122924
For Absolute Magnitudes = 0.00277565

The unbiased estimate of the standard deviation:
For Apparent Magnitudes = 0.837058
For Absolute Magnitudes = 0.397759

The maximum likelihood estimate of the standard deviation:
For Apparent Magnitudes = 0.829683
For Absolute Magnitudes = 0.394254

The unbiased estimate of the standard error (standard deviation) of the sample mean:
For Apparent Magnitudes = 0.110871
For Absolute Magnitudes = 0.0526844

The mean absolute deviation:
For Apparent Magnitudes = 0.651394
For Absolute Magnitudes = 0.262456

The median absolute deviation:
For Apparent Magnitudes = 0.5
For Absolute Magnitudes = 0.2

The inter-quartile range:
For Apparent Magnitudes = 1.045
For Absolute Magnitudes = 0.42

The Quartiles:
For Apparent Magnitudes = {6.335, 6.68, 7.38}
For Absolute Magnitudes = {4.94, 5.17, 5.36}

The coefficient of skewness:
For Apparent Magnitudes = -0.463522
For Absolute Magnitudes = 1.64722

Pearson's first coefficient of skewness:
For Apparent Magnitudes = {3.21301, 1.45686, 1.09846, 0.990939, -0.406813, -0.765211}
For Absolute Magnitudes = {2.01127, 1.86042, 0.57824, 0.351972, -0.100564}

Pearson's second coefficient of skewness:
For Apparent Magnitudes = 0.274143
For Absolute Magnitudes = 0.125704

The quartile coefficient of skewness:
For Apparent Magnitudes = 0.339713
For Absolute Magnitudes = -0.0952381

The kurtosis coefficient:
For Apparent Magnitudes = 3.52308
For Absolute Magnitudes = 9.01574

The kurtosis excess:
For Apparent Magnitudes = 0.523079
For Absolute Magnitudes = 6.01574
The confidence interval of the population mean based on the normal distribution:
For Apparent Magnitudes = \{6.53919, 6.97379\}
For Absolute Magnitudes = \{5.08341, 5.28993\}

The confidence interval for the population variance based on the chi-square distribution:
For Apparent Magnitudes = \{0.499411, 1.05444\}
For Absolute Magnitudes = \{0.112768, 0.238094\}

B.2 : Graphical Representations

The Histograms of the Apparent and Absolute Magnitudes of the stellar Group

B.3: Optimum Choice of \( \sigma \)

Graphical representations of the percentage errors for the Mean and Median

The minimum value of the percentage error in the Mean is = 0.663442 occurs at \( \sigma = 0.98 \)
The minimum value of the percentage error in the Median is = 0.765067 occurs at \( \sigma = 1.04 \)
The optimum value of \( \sigma \) is = 1.01

B.4: Distance Analysis

The value of \( m_1 \) is = 9.99376
The value of \( M \) is = 8.15025
The value of \( \sigma \) is = 1.01

The value of \( M_0 \) is = 6.59644
The value of \( \alpha \) is = 1.82526
The value of \( y \) is = 1.73244
The accuracy of the computed value of $y$ is $8.76278 \times 10^{-7}$.

The distance $r$ of the stellar group as obtained from the statistical method is 21.3555 parsec.

The Median of the individual distances is 20.8333 parsec.

The Mode of the individual distances is $\{23.2558, 25.0, 30.303\}$ parsec.

The geometric mean of the individual distances is 20.6050 parsec.

The harmonic mean of the individual distances is 19.1706 parsec.

The mean value of the individual distances is 21.9013 parsec.

The percentage error between $r$ and $\bar{r}$ is 2.4917.

B. 5: Frequency functions of the absolute and apparent magnitudes for The G5V stars

$\Phi(M) = 0.398992 \times e^{-0.490148 (-6.59644+M)^2}$

$\Psi(m) = 0.398992 \times e^{-0.490148 (-8.244+m)^2}$