Greener Journal of Physics and Natural Sciences
Vol. 4(1), pp. 07-12, 2019
ISSN: 2384-6410

DOI Link: https://doi.org/10.15580/GJPNS.2019.1.102619194
http://gjournals.org/GJPNS

Theorem for Change of Generalized Impulse in Theoretical Mechanics

Anastas Ivanov Ivanov

Todor Kableshkov University of Transport, Sofia, Bulgaria

ARTICLE INFO

Article No.:102619194
Type: Research
DOI:10.15580/GJPNS.2019.1.102619194

Submitted: 26/10/2019
Accepted: 29/10/2019
Published: 22/11/2019

**Corresponding Author*
Anastas Ivanov
E-mail: aii2010@abv.bg

Keywords: Rigid Body Mechanics; absolute general motion; additional kinetic characteristics; theorem; generalized impulse

ABSTRACT

In this article, a rigid body absolute general motion is studied. The rigid body is assumed as homogeneous and unsymmetrical. New additional kinetic characteristics are used. The most important of these are the following: the vector-real generalized velocity of the rigid body, the vector-generalized impulse of the rigid body and the vector-real generalized force of the rigid body. Using these new kinetic characteristics, a new theorem is defined. It is called Theorem for change of the rigid body generalized impulse. With its help, the differential equations in matrix form are obtained. These equations describe the rigid body absolute general motion. Two cases are studied - a pole that coincides and does not coincide with the rigid body mass center. The new theorem enriches the theory of Rigid Body Mechanics. The compact matrix equations provide an excellent opportunity for studying the most complicated dynamic models. They are suitable for numerical solutions with modern mathematical programs such as MatLab, MathCAD and others.

1. INTRODUCTION

During the Renaissance period of the evolution of science, Mathematics and Mechanics with all of their branches, such as Rigid Body Mechanics (Goldstein et al., 2000), Mechanics of Material (Timoshenko, 2002), Fluid Mechanics (Krause, 2005), Analytical Mechanics (Pars, 1964), and others, have been developing in parallel and jointly. The mathematical apparatus, that serves the various branches of Mechanics, is complicated. The equations become long and complex. This necessitates the introduction of many new abbreviated forms of mathematical recordings, such as “Nabla operator”, “Gradient”, ”Divergence”, “Curl”, “Convective derivative”, “Laplace operator”, “Tensor derivative” and others. Nevertheless, during this period, Mechanics remains in scalar-vector form. In 1857, British mathematician Arthur Cayley (1821-1895) published his treatise “A memoir on the theory of matrices” (Cayley, 1858). So, he gave a powerful weapon in the hands of scientists. Matrices and matrix calculus are used in Mechanics later after the development of one of the most powerful method – Finite Element Method (Kazakov., 2010; Handruleva et al., 2012), as well as when numerical methods and electronic computings are being developed (Karamanski, 1976). It turns out that many branches of Mechanics, and mainly Rigid Body Mechanics, remain preserved in their original classical form, namely in scalar-vector form, for example (Yablonsky, 1984, a, b).

This article shows in a briefly and essential form a new theorem called Theorem for change of the rigid body generalized impulse. It has already been used in some author's publications, for example (Ivanov, 2017, a, b; Ivanov, 2018, a, b). Here, the title of this paper coincides with the name of that theorem. In this way, the main purpose of the study is realized, namely, it has to promote this very interesting and useful theorem.

2. ADDITIONAL KINETICAL CHARACTERISTICS

Two ideal rigid bodies $"A"$ and $"B"$ are studied, (Fig. 1).

The body $"A"$ is considered absolutely motionless. The body $"B"$ is free and it performs a general motion. The coordinate system $N\xi\eta\zeta$ is connected to body $"A"$. The coordinate system $OXYZ$ is connected to the pole O and it performs a translational motion. The coordinate system $Oxyz$ is constantly connected to the pole O and to the body $"B"$. All vectors and matrices, which are referenced to the coordinate systems $N\xi\eta\zeta$ and $OXYZ$, are indicated by a lower index $"A"$ and when they are referenced to the coordinate system $Oxyz$, they are indicated by a lower index $"B"$. The spherical component of the rigid body motion is described by Cardanangles ψ, θ and φ. The law of rigid body motion is given by the vector of generalized coordinates:

(2.1) $$\mathbf{q} = \langle \xi_O \quad \eta_O \quad \zeta_O \quad \psi \quad \theta \quad \varphi \rangle^T .$$

The body $"A"$ is assumed as homogeneous and unsymmetrical.

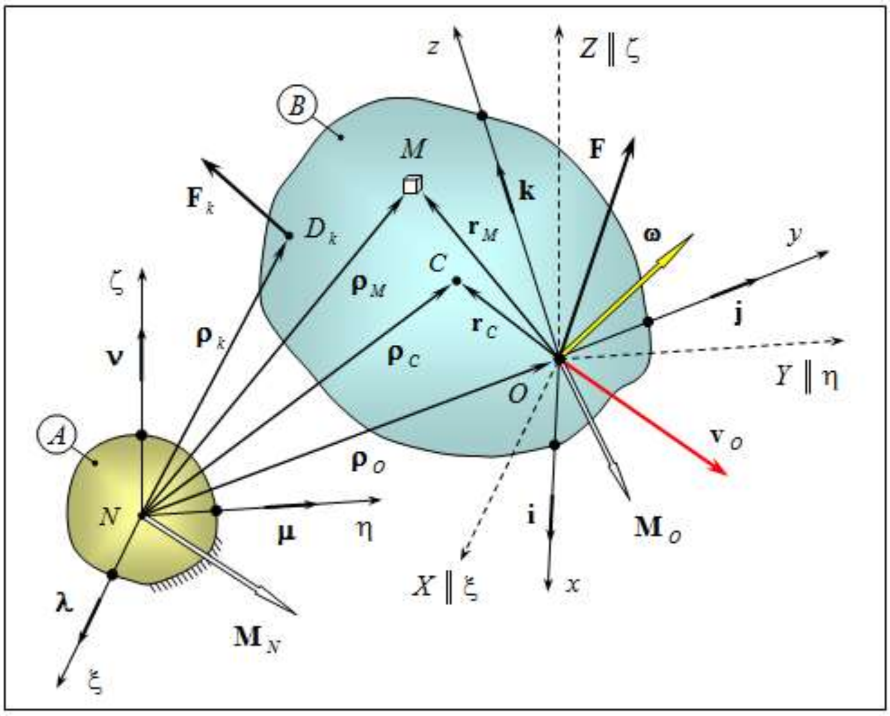

Fig.1: Dynamics of rigid body absolute general motion

A **vector-*real* generalized velocity** of the rigid body is defined. It refers to a random chosen pole O from the body and to a coordinate system $N\xi\eta\zeta$ or $OXYZ$:

(2.2) $$\mathbf{u}_{O,A} = \begin{bmatrix} \mathbf{v}_{O,A} \\ \boldsymbol{\omega}_A \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{\rho}}_{O,A} \\ \boldsymbol{\omega}_A \end{bmatrix} = \langle \dot{\xi}_O \quad \dot{\eta}_O \quad \dot{\zeta}_O \quad \omega_X \quad \omega_Y \quad \omega_Z \rangle^T .$$

Similar **vector-*real* generalized velocity** of the rigid body is defined. It refers to a random chosen pole O from the body and to a coordinate system $Oxyz$:

(2.3) $$\mathbf{u}_{O,B} = \begin{bmatrix} \mathbf{v}_{O,B} \\ \boldsymbol{\omega}_B \end{bmatrix} = \left\langle \; v_{O,x} \quad v_{O,y} \quad v_{O,z} \quad \omega_x \quad \omega_x \quad \omega_x \; \right\rangle^T .$$

Two transition matrices $\mathbf{W}_{A,B}$ and $\mathbf{W}_{B,A}$ having a dimension 6×6 are constructed. They are composed by the basic transition matrices $\mathbf{U}_{A,B}$ and $\mathbf{U}_{B,A}$ with dimension 3×3:

(2.4) $$\mathbf{W}_{A,B} = \begin{bmatrix} \mathbf{U}_{A,B} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{A,B} \end{bmatrix}, \qquad \mathbf{W}_{B,A} = \begin{bmatrix} \mathbf{U}_{B,A} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{B,A} \end{bmatrix}.$$

The relationship between the vector-*real* generalized velocity $\mathbf{u}_{O,A}$ and the vector of generalized velocity $\dot{\mathbf{q}}$ is performed by the equations:

(2.5) $$\mathbf{u}_{O,A} = \mathbf{H}.\dot{\mathbf{q}}, \qquad \mathbf{H} = \begin{bmatrix} \mathbf{E} & \mathbf{0} \\ \mathbf{0} & \overline{\mathbf{H}} \end{bmatrix}, \qquad \mathbf{E} = \mathbf{diag}\,[\,1\,]_3, \qquad \mathbf{0} = [\,0\,]_{3\times3},$$

(2.6) $$\overline{\mathbf{H}} = \begin{bmatrix} 1 & 0 & \sin\theta \\ 0 & \cos\psi & -\cos\theta.\sin\psi \\ 0 & \sin\psi & \cos\theta.\cos\psi \end{bmatrix}.$$

A **vector-generalized impulse** of the rigid body is defined. This vector, for a random chosen pole O from the body, has the form:

(2.7) $$\mathbf{D}_{O,A} = \begin{bmatrix} \mathbf{P}_A \\ \mathbf{L}_{O,A} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{S}^T_{C,A} \\ \mathbf{S}_{C,A} & \mathbf{J}_{O,A} \end{bmatrix} . \begin{bmatrix} \mathbf{v}_{O,A} \\ \boldsymbol{\omega}_A \end{bmatrix} = \mathbf{A}_{O,A}.\mathbf{u}_{O,A},$$

(2.8) $$\mathbf{A}_{O,A} = \begin{bmatrix} \mathbf{M} & \mathbf{S}^T_{C,A} \\ \mathbf{S}_{C,A} & \mathbf{J}_{O,A} \end{bmatrix}.$$

The matrix $\mathbf{A}_{O,A}$ is described in the publication (Ivanov, 2017, a).

A **vector-generalized impulse** of the rigid body for an immovable pole N is defined, (Fig.1). This vector is also linked indirectly with the pole O and with the mass center C of the body:

(2.9) $$\mathbf{D}_{N,A} = \begin{bmatrix} \mathbf{P}_A \\ \mathbf{L}_{N,A} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{S}^T_{C,A} \\ \mathbf{S}_{C,A} & \mathbf{J}_{O,A} \end{bmatrix} . \begin{bmatrix} \mathbf{v}_{O,A} \\ \boldsymbol{\omega}_A \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_{O,A} & \mathbf{0} \end{bmatrix} . \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{C,A} \end{bmatrix} . \begin{bmatrix} \mathbf{v}_{C,A} \\ \boldsymbol{\omega}_A \end{bmatrix},$$

(2.10) $$\mathbf{D}_{N,A} = \mathbf{A}_{O,A}.\mathbf{u}_{O,A} + \overline{\mathbf{T}}_{O,A}.\mathbf{A}_{C,A}.\mathbf{u}_{C,A} = \mathbf{D}_{O,A} + \overline{\mathbf{T}}_{O,A}.\mathbf{D}_{C,A},$$

(2.11) $$\overline{\mathbf{T}}_{O,A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_{O,A} & \mathbf{0} \end{bmatrix},$$

(2.12) $$\tilde{\mathbf{R}}_{O,A} = \begin{bmatrix} 0 & -\zeta_O & \eta_O \\ \zeta_O & 0 & -\xi_O \\ -\eta_O & \xi_O & 0 \end{bmatrix}.$$

A **vector-*real* generalized force** of the rigid body is defined. This vector, for a random chosen pole O from the body, has the form:

(2.13) $$\mathbf{Q}_{O,A} = \begin{bmatrix} \mathbf{F}_A \\ \mathbf{M}_{O,A} \end{bmatrix}.$$

Another **vector-*real* generalized force** of the rigid body for an immovable pole N is defined. This vector is also linked indirectly with the pole O of the body:

(2.14) $$\mathbf{Q}_{N,A} = \begin{bmatrix} \mathbf{F}_A \\ \mathbf{M}_{N,A} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_A \\ \mathbf{M}_{O,A} + \tilde{\mathbf{R}}_{O,A}.\mathbf{F}_A \end{bmatrix} = \begin{bmatrix} \mathbf{F}_A \\ \mathbf{M}_{O,A} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_{O,A} & \mathbf{0} \end{bmatrix} . \begin{bmatrix} \mathbf{F}_A \\ \mathbf{M}_{O,A} \end{bmatrix},$$

(2.15) $$\mathbf{Q}_{N,A} = \mathbf{Q}_{O,A} + \overline{\mathbf{T}}_{O,A}.\mathbf{Q}_{O,A}.$$

3. THEOREM OF CHANGE OF RIGID BODY GENERALIZED IMPULSE

The theorem states: *The first time derivative of the rigid body generalized impulse for a fixed pole is equal to its real generalized force determined for that pole.*

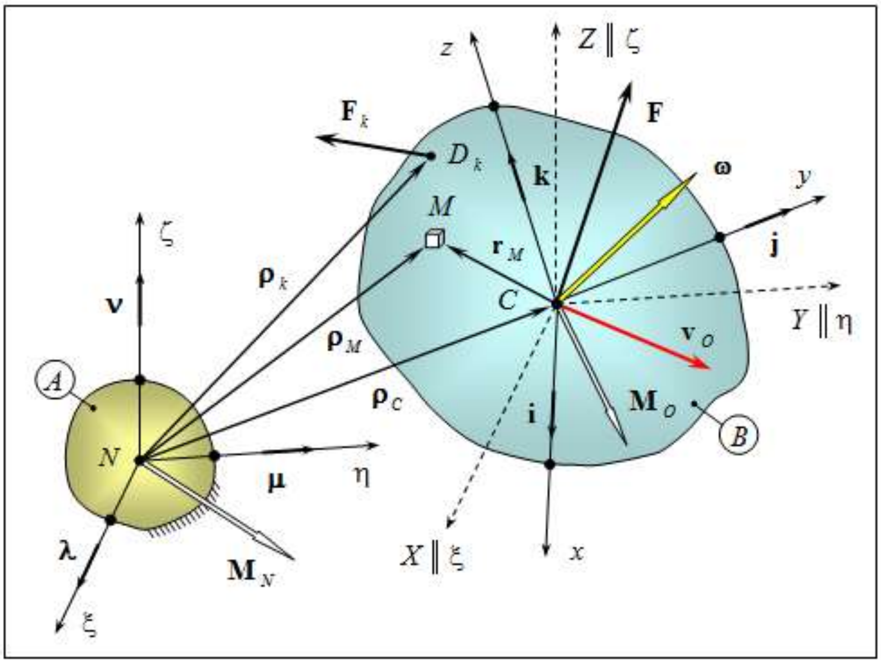

Fig.2: Dynamical model where the pole O coincides with the mass center C

The mathematical record of the stated above theorem has the form:

(3.1) $$\frac{d\,\mathbf{D}_{N,A}}{dt} = \mathbf{Q}_{N,A},$$

(3.2) $$\frac{d}{dt}\left(\mathbf{A}_{O,A}.\mathbf{u}_{O,A} + \overline{\mathbf{T}}_{O,A}.\mathbf{A}_{C,A}.\mathbf{u}_{C,A} \right) = \mathbf{Q}_{O,A} + \overline{\mathbf{T}}_{O,A}.\mathbf{Q}_{O,A},$$

(3.3)
$$\mathbf{A}_{O,A}.\dot{\mathbf{u}}_{O,A} + \dot{\mathbf{A}}_{O,A}.\mathbf{u}_{O,A} + \dot{\overline{\mathbf{T}}}_{O,A}.\mathbf{A}_{C,A}.\mathbf{u}_{C,A} +$$
$$+\begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_{O,A}.\mathbf{M}.\dot{\mathbf{v}}_{C,A} \end{bmatrix} = \mathbf{Q}_{O,A} + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_{O,A}.\mathbf{F}_{A} \end{bmatrix}.$$

Now, the theorem of the mass center motion is used:

(3.4)
$$\mathbf{M}.\dot{\mathbf{v}}_{C,A} = \mathbf{F}_{A}.$$

Through this theorem equation (3.3) takes the following form:

(3.5)
$$\mathbf{A}_{O,A}.\dot{\mathbf{u}}_{O,A} + \dot{\mathbf{A}}_{O,A}.\mathbf{u}_{O,A} = \mathbf{Q}_{O,A} - \dot{\overline{\mathbf{T}}}_{O,A}.\mathbf{A}_{C,A}.\mathbf{u}_{C,A}.$$

Let us assume the pole O coincides with the mass center C of the body, (Fig.2).
Then equation (3.5) take the following form:

(3.6)
$$\mathbf{A}_{C,A}.\dot{\mathbf{u}}_{C,A} + \dot{\mathbf{A}}_{C,A}.\mathbf{u}_{C,A} = \mathbf{Q}_{C,A}.$$

The equation (3.6) is written shortly by the following manner:

(3.7)
$$\frac{d\,\mathbf{D}_{C,A}}{dt} = \mathbf{Q}_{C,A}.$$

Equation (3.7) performs the Theorem for change of the rigid body generalized impulse towards the movable pole O, which coincides with the mass center C. This variant of the theorem is speaking that way: *The first time derivative of the rigid body generalized impulse for the body mass center is equal to its real generalized force determined for that center.*

4. CONCLUSION

Some new kinetic characteristics have been introduced.

The most important characteristics are the vector-*real* generalized velocity and the vector-generalized impulse for an ideal rigid body.

A new theorem, called *Theorem for change the rigid body generalized impulse* for the fixed pole N or for the movable pole O coinciding with the mass center C is defined. It is applied to study the absolute general motion of a free asymmetric ideal rigid body.

The stated theorem is formulated directly. Nevertheless, it represents a summary of the two main theorems in Dynamics: Theorem for change of linear momentum and Theorem for change of angular momentum, applied to the studied rigid body.

The directly defining of that new theorem became possible thanks to introducing the new kinetical characteristics and using the matrix operations.

REFERENCES

CayleyA. A memoir on the theory of matrices, Philosophical Transactions of the Royal Society of London, vol. 148, 1858, 17-37.

Goldstein H., C. Pole, J. Safko, Classical Mechanics. Columbia Univ., Univ. of South Carolina, 2000, 646 p.

Handruleva A.K., V.D. Matuski, K.S.Kazakov, Modeling of building constructions with program product SAP 2000, VSU “LyubenKaravelov”, Sofia, 2012, 365 p. (in Bulgarian)

Russian)

Ivanov A.I., Theoretical Matrix Study of Rigid Body General Motion. Greener Journal of Physics and Natural Sciences, vol. 3 (2), 2017, 009-020.

Ivanov A.I., Theoretical Matrix Study of Rigid Body Pseudo Translational Motion. Greener Journal of Physics and Natural Sciences, vol. 3 (2), 2017, 021-031.

Ivanov A.I., Theoretical matrix study of rigid body relative motion. International Journal of Advancement in Engineering Technology, Management and Applied Science (IJAETMAS), Vol. 05, Issue 05, 2018, 21-28.

Ivanov A.I., Theoretical Matrix Study of Rigid Body Absolute Motion. The International Journal of Engineering and Science (IJES), Vol. 7, Issue 6, Ver. II, 2018, 01-08.

Karamanski T.D., Numerical methods in Structural Mechanics, Technika, Sofia, 1976, 528 p. (in Bulgarian)

Kazakov K.S., Finite Element Method for modeling of building constructions, Academic publishing house "Prof. Marin Drinov", Sofia, 2010, 518 p. (in Bulgarian)

Krause E., Fluid Mechanics, Springer-Verlag, Berlin, Heidelberg, 2005, 363 p.

Pars L.A., A Treatise on Analytical Dynamics, Heineman, London, 1964, 636 p.

Timoshenko S.P., J.M. Gere, Mechanics of Matrials, Lan, Sankt Petersburg, 2002, 670 p. (in Russian).

Yablonsky A.A., Course of Theoretical mechanics. vol. 1, "Vishayashkola, Moscow, 1984, 344 p. (in Russian)

Yablonsky A.A., Course of Theoretical mechanics. vol. 2, "Vishayashkola, Moscow, 1984, 424 p. (in Russian)

Cite this Article: Ivanov A (2019). Theorem for Change of Generalized Impulse in Theoretical Mechanics. Greener Journal of Physics and Natural Sciences, 4(1): 07-12, https://doi.org/10.15580/GJPNS.2019.1.102619194