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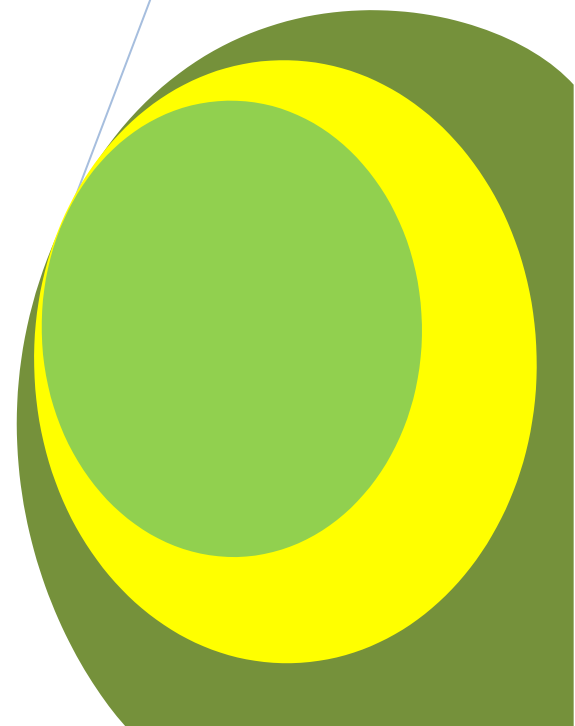
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Lifting Chandra Matrix to Solve Goldbach Conjecture

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Short Comm.

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ABSTRACT

This paper hints a very simple method of lifting Chandra matrices to solve Goldbach conjecture, by lifting we mean to add a positive integer to every element of the matrix, this way we constructed a set of Chandra matrices, which are equivalent to a progressive modulo operations until any large number, for judging if the number is a prime or not. The advantage of this method is that it offers a quick and an easy understandable way for any prime number verification or partition. The disadvantage of this method is that both the number of matrices and the dimension of each matrix are infinite. In this paper, we show that all positive even integers $n \geq 40$ can be expressed as the sum of two primes, those $n < 40$ are trivial cases.

Keywords: Chandra matrix; Goldbach conjecture; Prime numbers.

INTRODUCTION

Goldbach's original conjecture was written in June 7, 1742 letter to Euler, states "at least it seems that every number that is greater than 2 is the sum of three primes". Goldbach's conjecture is one of the oldest and best-known unsolved problems in number theory. Today it stands as: "Every even integer greater than 4 can be expressed as the sum of two primes" [1]. It seems that this conjecture was observed by Descartes even earlier, as remarked by Erdős, we shall continue to call this Goldbach's conjecture. The conjecture has been shown to hold up through 4×10^{18} [2] at least, but remains unproven in a strictly mathematical sense, despite considerable effort from researchers all over the world. The main reason is simple, because there are infinite numbers of numbers. For an engineer, any small number can be thought as a relatively big number; for mathematician, however, any big number is still a small number. Bearing this philosophy in mind, we declared following matrix based derivations, inspired by Chandra matrix, hope it will be beneficial to both engineers and scientists communities.

Note that here Goldbach considered the number 1 to be a prime, a convention that is no longer followed. As re-expressed by Euler, an equivalent form of this conjecture: Every even integer greater than 4 can be expressed as the sum of two primes. There are a number of ways to approach this conjecture, the most authentic way is to use the circle method [3], which is hard to understand; the next practical method is to use the Sieve [4], still not that easy to get it, another way is to use probabilistic analysis [5], preferred more by engineer; the modern one is using the matrix [6], which is very intuitive; anyway, what we are interested in is some construction of relatively big numbers, that are used to manipulate the encryption key [7], as such we focus on the latest matrix method.

Main Derivations

Let us look at a matrix: in 1934, one ground breaking mathematician from the East Indian (now Bangladesh) Harish Chandra (1923–1983), in the field of number theory, has made a founding contributions of harmonic analysis on semisimple Lie groups. This subject is equally important for an engineer, as a synthesis of Fourier analysis, special functions and invariant theory etc., and it has also become a basic tool in analytic number theory, via the theory of automorphic forms, leading to Langlands program eventually. It became one of the major mathematical edifices of the second half of the twentieth century [8], after World War II.

Chandra matrix is a square sieve, where the first row of the square sieve consists of the first element of 4, the difference between next every two adjacent numbers is 3, forms an arithmetic sequence: 4,7,10, ... The first column equals to the first row. The second row, third row, any subsequent rows are also arithmetic sequence, but the difference between two adjacent numbers gradually become larger, and they are 5,7,9,11,13, ... respectively, and they are all odd numbers, and the matrix is symmetrical, as shown below with modulo equivalent representation:

4	7	10	13	16	19	22	25	i.e. mod(n,3)=1
7	12	17	22	27	32	37	42	i.e. mod(n,5)=2
10	17	24	31	38	45	52	59	i.e. mod(n,7)=3
13	22	31	40	49	58	67	76	i.e. mod(n,9)=4
16	27	38	49	60	71	82	93	i.e. mod(n,11)=5
19	32	45	58	71	84	97	110	i.e. mod(n,13)=6

The secret of this square sieve is: If a natural number N appear in the table, then $2N+1$ certainly is not a prime number, because 2 times of remaining plus 1 is at least one of the divisors in the above modulo operations. If N does not appear in the table, then $2N+1$ is definitely a prime number, because 2 times of remaining plus 1 is not any of the divisor in the above modulo operation. Primes are left out, when we look at the first row, it sounds like $2/3$ of the numbers are left out, i.e. $\text{mod}(n,3)=0$ or 2 , and for the second row, $4/5$ of the numbers are left out, i.e. $\text{mod}(n,5)=0, 1, 3, 4$, and so on so forth, however, since the matrix is symmetrical, or triangularly shifted, there is an overlap, as such the gaps are made up by more and more next rows. That's why we have infinite primes, but less and less as the number becomes big.

Let's look at a few examples. Beginning with 4, skipped three numbers 1,2,3, of course, they will never appear in the table. Then, $2 \times 1 + 1 = 3$, $2 \times 2 + 1 = 5$, $2 \times 3 + 1 = 7$. You see, 3, 5, 7 are prime numbers. Look at the number 17, 17 is in the symmetric matrix $\cdot 17^2 + 1 = 35$ and 35 is a prime number. Almost all primes can be launched from this table, assume that the number of primes follow the prime number theorem $x/\ln(x)$ in arithmetical range of the numbers.

Based on above observations, we made a few similar matrices accordingly (lifting by 1):

5	8	11	14	17	20	i.e. mod(n,3)=2
8	13	18	23	28	33	i.e. mod(n,5)=3
11	18	25	32	39	46	i.e. mod(n,7)=4

If natural number N in the matrix, then $2N-1$ is certainly not a prime number, if not, $2N-1$ is a prime number, because 2 times of remaining minus 1 is the divisor now, say, in the first matrix 5 is not appear, and 6 does not appear in the second matrix, $2^*6-1=2^*5+1=11$, so it was established.

Similarly, then set out a matrix (lifting by 2):

6	9	12	15	18	i.e. mod(n,3)=3
9	14	19	24	29	i.e. mod(n,5)=4
12	19	26	33	40	i.e. mod(n,7)=5

If the natural number N can be drawn from this matrix, 2^*N-3 is certainly not a prime number, if not $2N-3$ is a prime number, the reason is the same as stated above by the modulo representations.

Also listed (lifting by x):

4+x	7+x	10+x	13+x	i.e. mod(n,3)=1+x
7+x	12+x	17+x	22+x	i.e. mod(n,5)=2+x
10+x	17+x	24+x	31+x	i.e. mod(n,7)=3+x

Lifting by any positive integer can be obtained if the natural number N in the matrix, $2^*N-(2x-1)$ is certainly not a prime number, if not $2^*N-(2x-1)$ must be a prime number. Beginning with 5's matrix, natural number N does not appear, then $2N-1$ is a prime number. Matrix beginning with 6's, natural number N does not appear, then $2N-3$ is a prime number; the beginning of 7's,8's,9's, etc., the natural number N does not appear in these matrix, then $2N-5$, $2N-7$, $2N-9$, $2N-11$, etc. are prime numbers.

Notice that minuend $2N$ is an even number, and the subtrahend are all odd numbers, now let's only consider the lifted matrices that corresponding to the prime number $2x-1$, skip all others, then $2N$ can be expressed as a sum of two prime numbers, in another word:

There are four kinds of situations:

- Case A: $2N$ -prime = prime number
- Case B: $2N$ -prime = composite number
- Case C: $2N$ -composite number = prime number
- Case D: $2N$ -composite number = composite number

Case B and Case C can be transformed into each other, because $a-b=c$, then the $a-c=b$, the N in B is the numbers which appear in the matrix beginning of minus prime numbers, the N in C is the numbers not appear in the matrix beginning of minus composite numbers, the two N are of the same family, that is to say all the numbers which appear in the matrix beginning of minus prime numbers are the same as the numbers not appear in the matrix beginning of minus composite numbers, then all the numbers which not appear in the matrix beginning of minus prime numbers is same as the numbers appear in the matrix beginning of minus composite number, so the N in A and D is of same family as well.

Based on Chen's theorem [4], the family of BC goes all over the entire integer range until infinite, by using Goldbach partition pyramid shown in Figure 1, it is not hard to prove that the family AD sit right in between BC, as such, both families should have the comparable size, more over, the case A also sit in between case D, as such, $2N$ will cover all even numbers above 40, as shown below.

For example, all the even numbers not less than 40 can be expressed as an odd composite number plus an odd composite number, reason, due to the end of n must be 0,2,4,6,8, if we choose $\text{mod}(n,10)$:

- (1) If the end is 0, then $n = 15 + 5k$ ($k \geq 5$ is odd);
- (2) If the end is 2, then $n = 27 + 5k$ ($k \geq 3$ is odd);
- (3) If the end is 4, then $n = 9 + 5k$ ($k \geq 7$ is odd);
- (4) If the end is 6, then $n = 21 + 5k$ ($k \geq 5$ is odd);
- (5) If the end is 8, then $n = 33 + 5k$ ($k \geq 3$ is odd);

In summary, any even number not less than 40 can be expressed as an odd composite number plus an odd composite number. So the N in D include all even numbers big than 20, equivalently the N in A also contains all the numbers, A; $2N$ - prime number = prime number then $2N$ can be expressed as the sum of two prime numbers, so all the even numbers big than 40 can be expressed as the sum of two prime numbers, and the even numbers less than 40 can use the method of enumeration shown in Figure 1 as well.

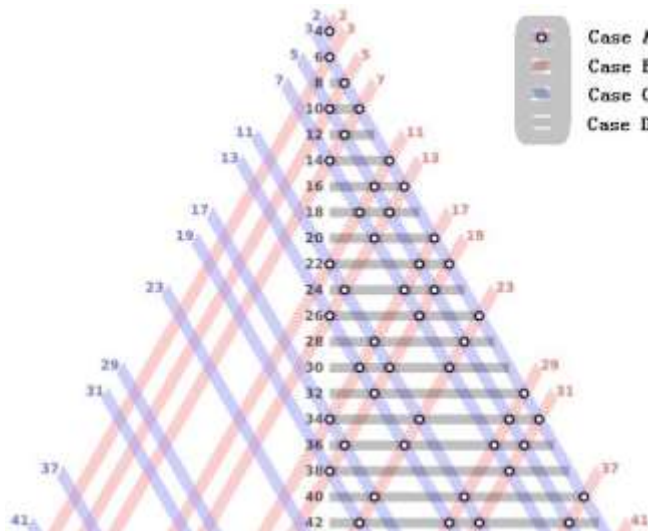


Figure 1: Goldbach Partition Pyramid

CONCLUSION

All the even numbers great than 40 can all be expressed as the sum of two prime numbers, finding these primes is hard by using hand calculation though, as such, we made a program in Matlab to compute that. This matrix based method will be supported with the Matlab program available on Matlab server, showing that the any even number can be split into two prime numbers.

AUTHOR CONTRIBUTIONS

Mr. Mi Zhou conceived and derived the original work that led to this submission, responded many comments from experts in the number theory in the past three years, he made the corrections, and played an important role in completing the final results. Prof. Steed Huang contributed to polishing and preparing of the manuscript, adding introduction, modulo representation and references, besides providing the guidance and encouragement.

REFERENCES

- [1] Eric W. Weisstein, "Goldbach Number", MathWorld; <http://mathworld.wolfram.com/GoldbachNumber.html>
- [2] Tomás Oliveira e Silva, "Goldbach conjecture verification", <http://sweet.ua.pt/tos/goldbach.html>
- [3] Jeff Law, "The Circle Method on the Binary Goldbach Conjecture", 36 Pages Report, Mathematics Department, Princeton University, April 3, 2005.
- [4] Jingrun Chen, "On the representation of a large even integer as the sum of a prime and the product of at most two primes". Kexue Tongbao 11 (9): 385–386, 1966.
- [5] Henry F. Fliegel; Douglas S. Robertson, "Goldbach's Comet: the numbers related to Goldbach's Conjecture", Journal of Recreational Mathematics, v21(1) 1-7, 1989.
- [6] Roger Ellman, A PROOF OF "GOLDBACH'S CONJECTURE", May 10, 2000, Roger Ellman, 320 Gemma Circle, Santa Rosa, CA 95404, USA.
- [7] Subhash Kak, Goldbach Partitions and Sequences, Resonance, Volume 19, Issue 11, pp 1028-1037, November 2014,.
- [8] Roger Howe, "A Biographical Memoir for Harish Chandra", National Academy of Sciences, 2011.

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